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PERFORMANCE LIMITS OF MULTISTAGE THERMOELECTRIC DEVICES

Irving I. Sochard

28 July 1961

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ORDNANCE CORPS • DEPARTMENT OF THE ARMY
WASHINGTON 25, D. C.

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Irving I. Sochard

FOR THE COMMANDER:
Approved by

William Piper
for I. Rotkin
Chief, Laboratory 300

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ABSTRACT

A simplified method of calculating the performance limits of multistage thermoelectric devices is presented and the results obtained for the multistage devices are compared with those for single-stage devices. The results indicate that at present there is little advantage in using more than one stage in a thermoelectric generator. In the case of a thermoelectric refrigerator, however, a small feasible multistage device should be able to produce temperature reduction about twice as great as that obtainable with a single-stage device. Using presently available materials, this corresponds to a maximum temperature reduction approximating 135°C for the multistage device. It is also shown that even if materials are substantially improved, it will probably never be feasible to construct a thermoelectric refrigerator that can cool to cryogenic temperatures while operating from room temperature ambient.

1. INTRODUCTION

A simplified method is presented for calculating the upper limit of performance obtainable from a given material or materials when used in a multistage thermoelectric device, or, conversely, the minimum material quality requirements for a given application. The basic arguments and equations are developed for both thermoelectric refrigerators and generators. The method of applying these equations for the case of a refrigerator is then considered in some detail and some general guides toward the design of a practical multistage cooling device are also given.

The maximum temperature difference, Δt_{max}, that can be produced by a single-stage thermoelectric refrigerator is given by

$$\Delta t_{max} = \frac{1}{2} Z t_c^2 \qquad \text{(ref 1)*} \qquad (1)$$

where t_c is the cold junction temperature. With the use of modern materials, this means a temperature reduction of 65° to 75° C can be achieved from room ambient temperature. If a further reduction in temperature is required, successive stages of refrigeration can be thermally cascaded. In this arrangement, each stage is used to lower the hot junction temperature of the one above it (fig. 1). The top stage extracts a quantity of heat, Q_c, from a load. Since each stage consumes some electric power, w_i, the quantity of heat it rejects is greater than the quantity of heat it absorbs by this amount. In addition, the amount of thermoelectric material needed in a stage is approximately proportional to the amount of heat to be handled by that stage, so the size of the successive stages is

* From reference 1, Z is the thermoelectric figure of merit and is defined as

$$Z = \left(\frac{\alpha_p - \alpha_n}{\sqrt{K_n \rho_p} + \sqrt{K_p \cdot \rho_n}} \right)^2$$

where d is the Seebeck coefficient, K is the thermal conductivity, and ρ is the electrical resistivity. The subscripts n and p refer to the semiconductor types in each leg of the thermoelectric couples.

progressively larger. Hypothetically, an arrangement such as that shown in figure 1 can be used to reach any desired temperature. In practice the quality of the material available places a lower limit on the temperature that can be achieved.

The advantage of cascading thermoelectric generators (fig. 2) is not so obvious as in the case of refrigerators. But it will be shown that it may in the future be possible to increase efficiency significantly by doing so.

2. INFINITE-STAGE THERMOELECTRIC REFRIGERATOR

The equation for the maximum coefficient of performance (cop) of a single-stage thermoelectric refrigerator operating through temperature interval Δt is given by

$$\mathrm{cop}_i = \frac{t_c}{\Delta t}\,\frac{\sqrt{1 + 1/2\,Z\,(t_h + t_c)} - \dfrac{t_h}{t_c}}{\sqrt{1 + 1/2\,Z\,(t_h + t_c)} + 1} \qquad \text{(ref 1)} \tag{2}$$

where cop_i is defined as the heat extracted by a stage over the work needed to operate that stage,

$$\mathrm{cop}_i = \left(\frac{q_c}{w}\right)_i , \tag{3}$$

and where Z is not a function of temperature.

Equation (2) can be rewritten as

$$\mathrm{cop}_i = \left(\frac{t}{\Delta t}\right)_i \epsilon \tag{2a}$$

where $\left(\frac{t}{\Delta t}\right)_i$ is the cop of an ideal or Carnot refriterator* and ϵ is a differential factor relating it to the cop of an actual refrigerator.

* In 1824 V.L.S. Carnot presented a paper on the thermal behavior of an ideal heat engine. This work later extended by Kelvin, showed there is a limit to the efficiency or coefficient of performance of even an ideal heat engine. Such a hypothetical device is called a Carnot refrigerator or engine. In exactly analogous fashion to the Carnot-Kelvin use of the ideal gas concept, we can here conceptually define an ideal thermoelectric material. Such a hypothetical material would have a Zt of infinity and an ϵ equal to one, and would give Carnot performance in any application.

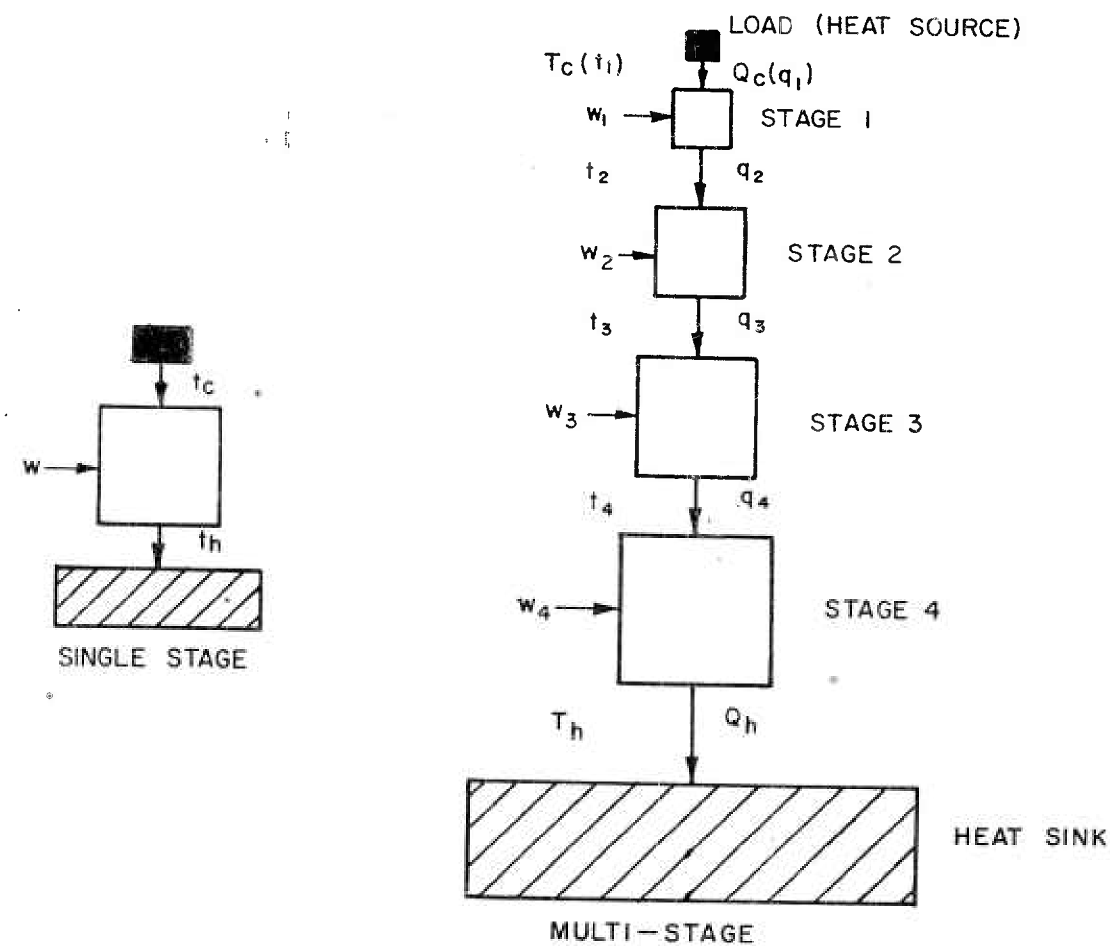

Figure 1. Block diagram of single and multistage thermoelectric refrigerator.

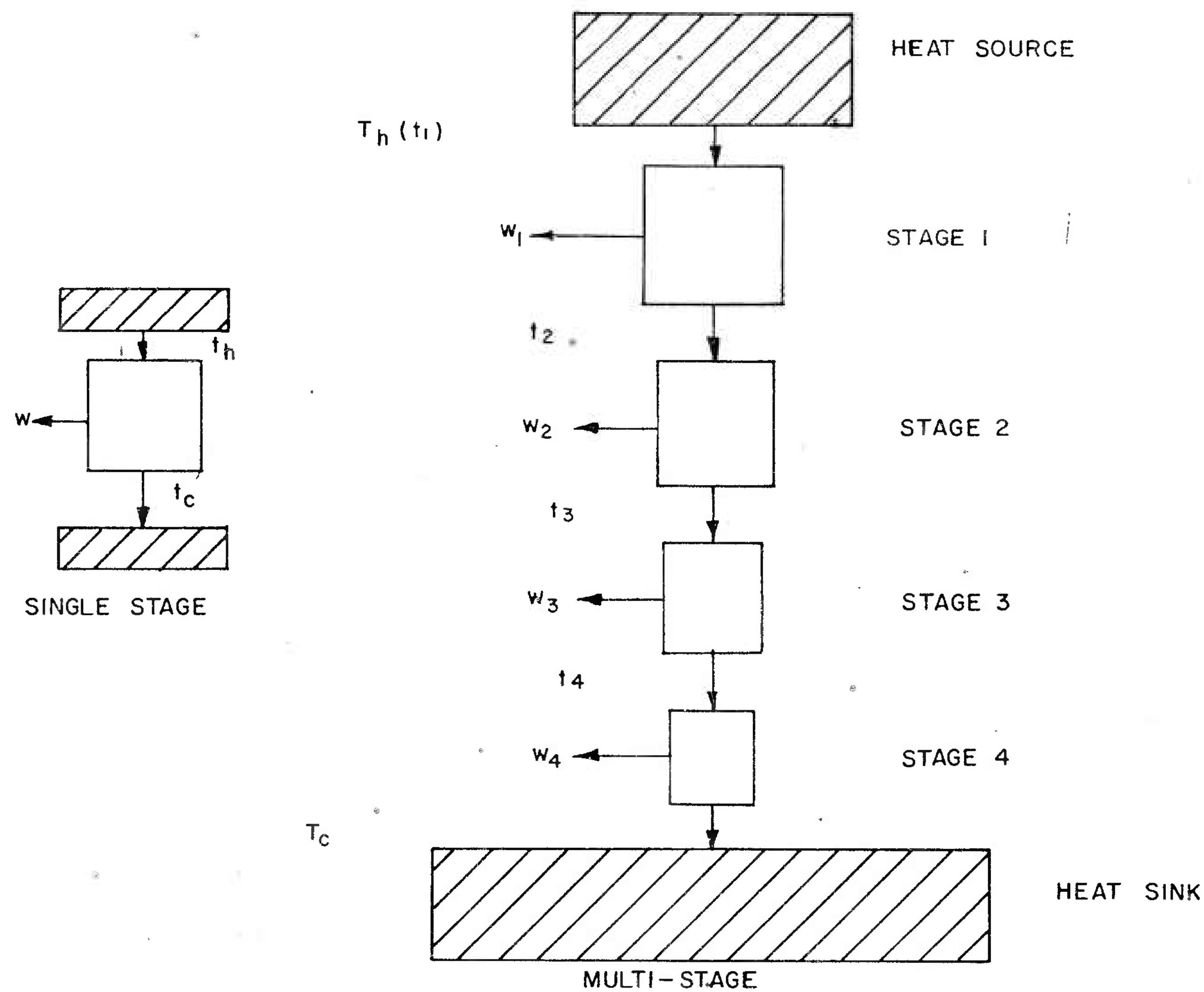

Figure 2. Block diagram of single and multistage thermoelectric generator.

The value of ϵ therefore ranges from 0 to 1. The value of ϵ is a maximum when Δt equals zero for any given value of Z and T. This value, ϵ_o is

$$\epsilon_o = \frac{\sqrt{1 + Zt} - 1}{\sqrt{1 + Zt} + 1} . \tag{4}$$

Therefore, an actual single-stage thermoelectric refrigerator most closely approximates a Carnot refrigerator when operating over a very small temperature interval.

It can be shown (ref 2) that the overall COP of a cascade of Carnot refrigerators is dependent only on the overall temperature covered and not on the number of stages. It is, therefore, reasonable to expect that the highest overall COP attainable from a chain of actual refrigerators covering a given temperature interval occurs when each one spans a vanishingly small temperature interval. This is verified in reference 3.

Equation (2a) for small Δt can now be rewritten as

$$\frac{q}{w} = \frac{t}{\Delta t} \epsilon_o \tag{2b}$$

Since, when Δt approaches 0, q is very much larger than w and $q_h = q_c + w$. In going to the limit of small Δt we get

$$q_c \rightarrow q \leftarrow q_h \; ; \; t_c \rightarrow t \leftarrow t_h \; ; \; w \rightarrow dq \text{ and } \Delta t \rightarrow dt .$$

Equation (2b) can be written as

$$\frac{q}{dq} = \frac{t}{dt} \epsilon_o \qquad \text{or} \qquad \epsilon_o \frac{dq}{q} = \frac{dt}{t} . \tag{2c}$$

Integrating this as follows:

$$\epsilon_o \int_{Q_c}^{Q_h} \frac{dq}{q} = \int_{T_c}^{T_h} \frac{dt}{t} , \tag{5}$$

where Q_c is the amount of heat entering at the cold end of the cascade and Q_h the amount leaving at the hot end, T_c is the temperature at which Q_c is absorbed, and T_h is the temperature at which Q_h is rejected. Then

$$\epsilon_o \ln \frac{Q_h}{Q_c} = \ln \frac{T_h}{T_c} . \tag{6}$$

This is valid for a range of temperature through which ϵ_o is sensibly constant. Taking antilogs of both sides we get

$$\frac{Q_h}{Q_c} = \left(\frac{T_h}{T_c}\right)^{1/\epsilon_o} \tag{7}$$

since

$$Q_c + W = Q_h, \quad \text{where} \quad W = \sum_i w_i,$$

and

$$\frac{Q_h}{Q_c} = \frac{Q_c + W}{Q_c} = 1 + \frac{1}{COP} = \left(\frac{T_h}{T_c}\right)^{1/\epsilon_o}, \tag{7a}$$

where COP is defined as

$$COP = \frac{Q_c}{W}, \tag{7b}$$

and finally

$$COP = \frac{1}{\left(\frac{T_h}{T_c}\right)^{1/\epsilon_o} - 1} \text{ *.} \tag{8}$$

3. INFINITE-STAGE THERMOELECTRIC GENERATOR

A similar derivation can be made for a cascade of thermoelectric generators as follows:

$$\eta = \frac{\Delta t}{t_h} \frac{\sqrt{1 + 1/2\, Z\, (t_h + t_c)} - 1}{\sqrt{1 + 1/2\, Z\, (t_h + t_c)} + \frac{t_c}{t_h}} \quad \text{(ref 1)} \tag{9}$$

where η is the single-stage efficiency. For small Δt

$$\eta = \frac{\Delta t}{t} \frac{\sqrt{1 + Zt} - 1}{\sqrt{1 + Zt} + 1} \tag{10}$$

which we write as

$$\eta = \frac{\Delta t}{t} \epsilon_o . \tag{11}$$

* Another approach and derivation of equation (8) may be found in reference 3.

Proceeding as above, we insert the definition of $\dot{\eta}$,

$$\frac{w}{q} = \frac{\Delta t}{t} \epsilon_o \quad , \tag{12}$$

since η is defined as the ratio of the amount of work produced to the heat entering the stage. Then, in the limit as Δt approaches zero, we have

$$\frac{dq}{q} = \epsilon_o \frac{dt}{t} \quad . \tag{13}$$

Integrating as before,

$$\int_{Q_c}^{Q_h} \frac{dq}{q} = \epsilon_o \int_{T_c}^{T_h} \frac{dt}{t} \quad , \tag{14}$$

we get

$$\ln \left(\frac{Q_h}{Q_c} \right) = \ln \left(\frac{T_h}{T_c} \right)^{\epsilon_o} \quad . \tag{15}$$

Taking antilogs of both sides,

$$\frac{Q_h}{Q_c} = \left(\frac{T_h}{T_c} \right)^{\epsilon_o} \quad \text{or} \quad \left(\frac{Q_c}{Q_h} \right) = \left(\frac{T_c}{T_h} \right)^{\epsilon_o} \quad ; \tag{16}$$

then

$$\frac{Q_c}{Q_h} = \frac{Q_h - W}{Q_n} = 1 - \underline{H} = \left(\frac{T_c}{T_h} \right)^{\epsilon_o} \tag{16a}$$

where

$$\underline{H} = \frac{W}{Q_h} \quad , \tag{16b}$$

giving

$$\underline{H} = 1 - \left(\frac{T_c}{T_h} \right)^{\epsilon_o} {}^{*} \quad . \tag{17}$$

* Another approach and derivation of equation (17) are found in references 3 and 4.

Equation (17) is valid over a temperature range in which ϵ_o is approximately constant. If ϵ_o varies slightly over this range, an average value of ϵ_o will give results accurate enough for most purposes. Throughout a rather wide temperature range extending up from normal room temperature, the values of ϵ_o obtained with the best currently available materials have tended to cluster around 0.17 (or $Zt = 1$). Figure 3 is a plot of maximum efficiency obtainable with material of this quality. Curves are plotted for a one-stage generator using equation (9), for a two-stage generator using equation (9) with η equal in both stages, and for an infinite-stage generator using equation (17). At present, thermoelectric materials of reasonable quality are available only up to about 1000° to 1200° K, so the advantage of multiple staging in thermoelectric generators is marginal since there will invariably be some loss between stages.

> NOTE: The significance of equations (7) and (16) is shown graphically in figure 4. The quantity of heat flowing in an infinite-stage thermoelectric cascade is shown as a function of temperature. The results are compared with that obtained for Carnot devices. The heat flux for each curve is arbitrarily equalized at T_o.

4. CALCULATIONS OF COEFFICIENTS OF PERFORMANCE WITH ϵ_o A FUNCTION OF TEMPERATURE

If ϵ_o is a function of temperature, equation (5) can be rewritten as

$$\int_{Q_c}^{Q_h} \frac{dq}{q} = \frac{1}{\epsilon_{o1}} \int_{T_c}^{T_1} \frac{dt}{t} + \frac{1}{\epsilon_{o2}} \int_{T_1}^{T_2} \frac{dt}{t} + \ldots + \frac{1}{\epsilon_{on}} \int_{T_{n-1}}^{T_h} \frac{dt}{t} \quad (18)$$

where T_c $(=T_o)$, T_1, T_2, T_3 ... T_i ... T_h $(=T_n)$ are the boundaries of n temperature regions over which ϵ_o can be considered sensibly constant at values ϵ_{o1}, ϵ_{o2}, ϵ_{o3} ... ϵ_{oi} ... ϵ_{on}. Because of the difficulty of measuring thermal conductivity accurately, present measurement techniques rarely permit the determination of ϵ_o more accurately than ± 10 percent and ϵ_o is not usually a rapid function of temperature. Therefore, the temperature intervals T_i to T_{i+1} can be of considerable size, if the value of ϵ_{oi} used is a reasonable average of ϵ_o for the temperature interval without significantly decreasing the accuracy of the results. Carrying out the integration, we get

$$\ln \frac{Q_h}{Q_c} = 1/\epsilon_{o1} \ln \frac{T_1}{T_c} + \frac{1}{\epsilon_{o2}} \ln \frac{T_2}{T_1} + \ldots + \frac{1}{\epsilon_{on}} \ln \frac{T_h}{T_{n-1}} \quad (19)$$

$$\frac{Q_h}{Q_c} = \left(\frac{T_1}{T_c}\right)^{1/\epsilon_{o1}} \left(\frac{T_2}{T_1}\right)^{1/\epsilon_{o2}} \ldots \left(\frac{T_h}{T_{n+1}}\right)^{1/\epsilon_{on}} \quad (20)$$

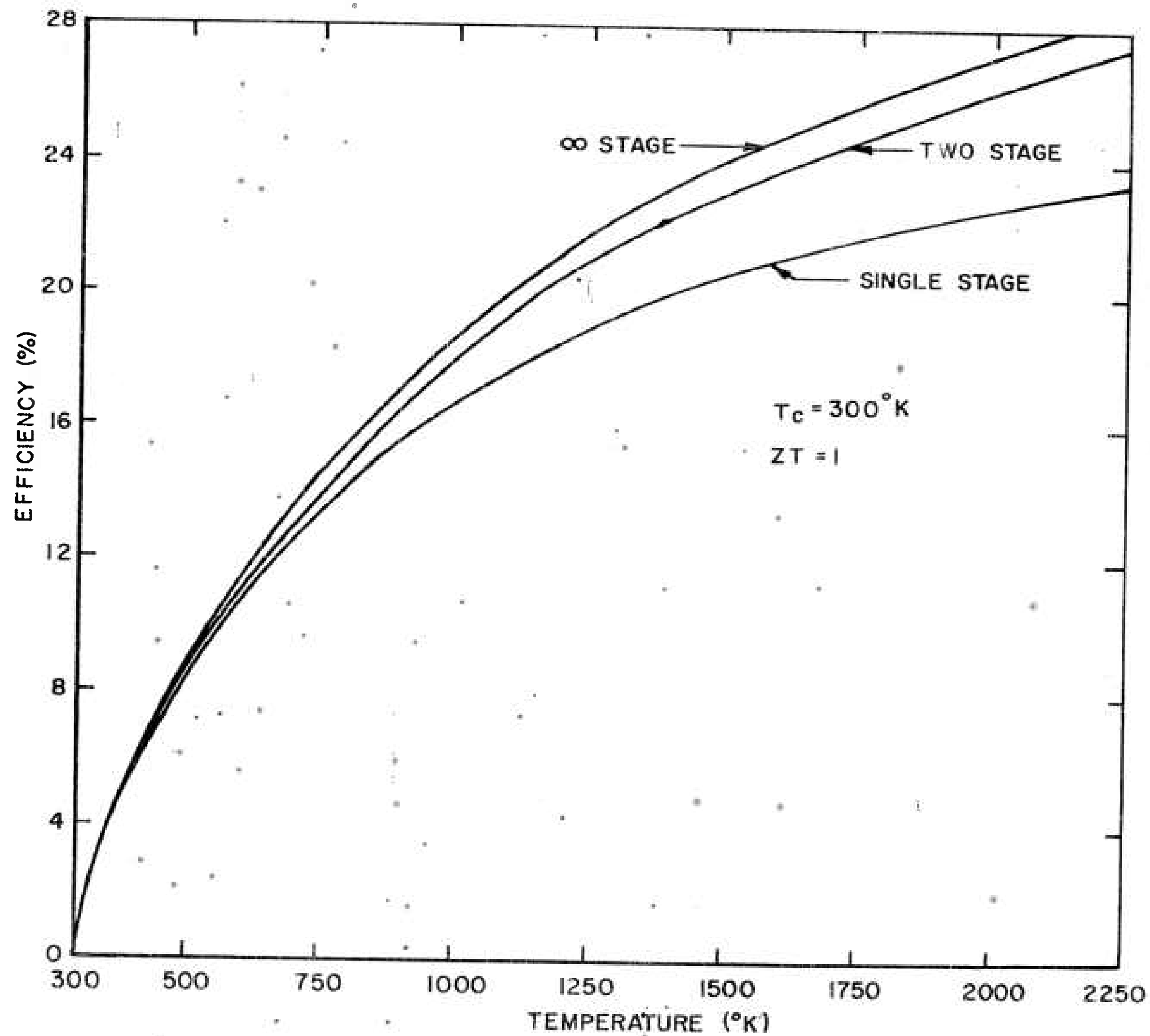

Figure 3. The maximum efficiency of one-, two-, and infinite-stage thermoelectric generators as a function of the hot-junction temperature.

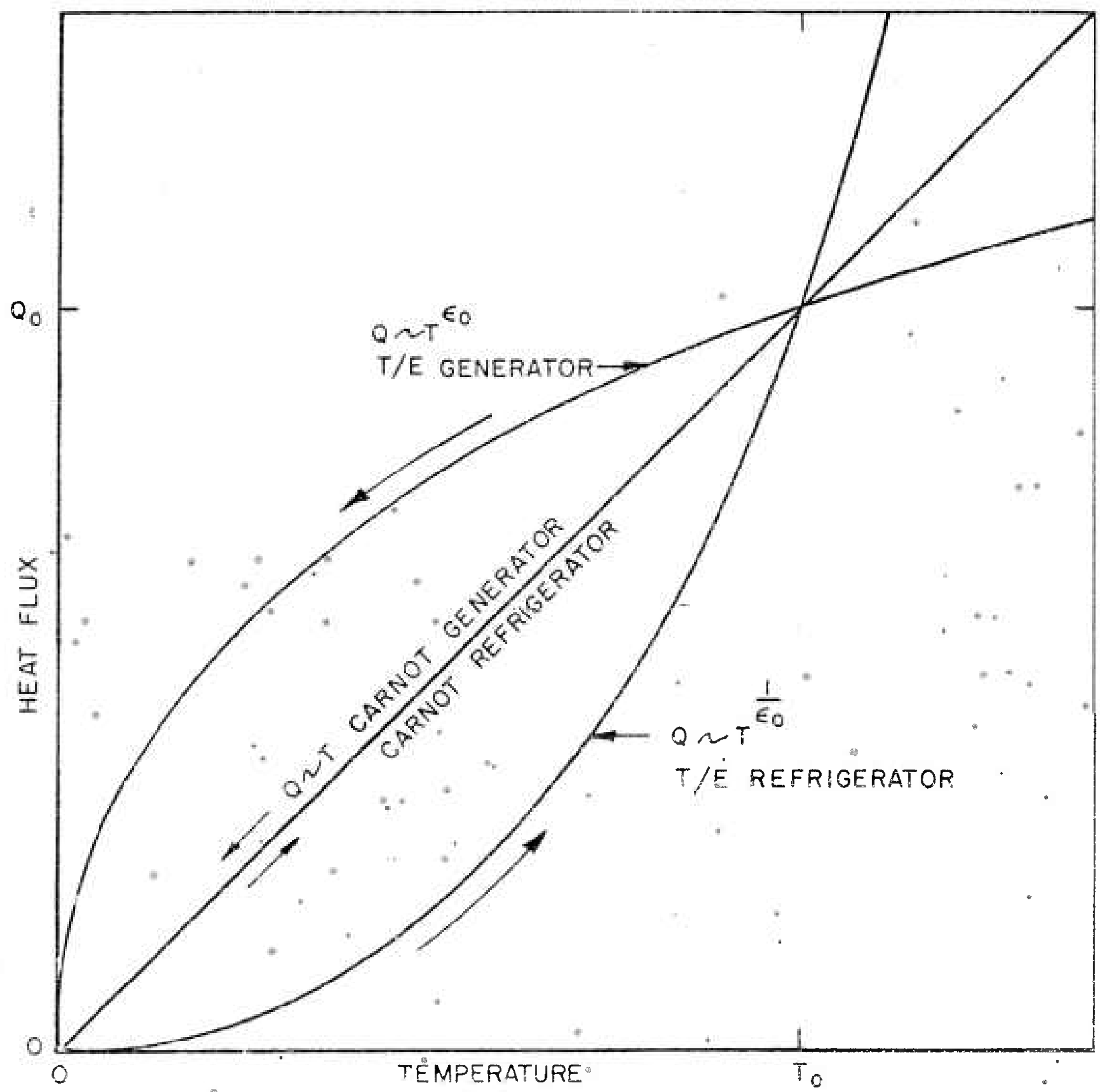

Figure 4. Heat flux in infinite-stage thermoelectric devices compared with the equivalent Carnot devices.

$$\mathrm{COP} = \frac{1}{\frac{Q_h}{Qc} - 1} = \frac{1}{\pi\left(\frac{T_n}{T_{n-1}}\right)^{1/\epsilon_{oi}} - 1} \quad (21)$$

An important advantage of the form of equation (20) is that, when the value of Q_h/Q_c has been computed for an interval $T_h - T_c$, the values at some intermediate temperatures are known as well. It is, therefore, possible to plot the minimum value of Q_h/Q_c (or the maximum value of COP) as a function of interval size if either T_h or T_c is fixed.

5. AN ILLUSTRATIVE CALCULATION OF COP WITH A VARIABLE ϵ_o

The thermoelectric properties of useful thermoelectric materials are usually given in terms of Z. Reference 5 gives Z as a function of temperature for many important materials. For simplicity, the following example will be performed on a hypothetical material or sequence of materials each employed in a specific temperature range (ref 6).

This material will be postulated to have a value of $Z = 3 \times 10^{-3}\,{}^{o}C^{-1}$ * over a temperature interval from room ambient (300°K) down (ref 5). For the temperature range from 200° to 300° K, this assumes a material about as good as the best presently available. Below 200°K, the assumption becomes progressively more futuristic.

Equation (20) can be modified as follows to reduce the amount of computation. The ratio T_i/T_{i-1} can be kept constant at some value R chosen so that

$$\frac{T_h}{T_c} = R^n \quad (22)$$

Then equation (20) can be rewritten as

$$\frac{Q_h}{Q_c} = R^{\left(\sum_{i=1}^{i=n} 1/\epsilon_{oi}\right)}, \quad (22a)$$

The ratio R must be small enough so that the limits stated on the variation of ϵ_o during an interval will hold throughout.

Since Z is known as a function of temperature, the first numerical step will be to compute $1/\epsilon_o$ as a function of temperature, using equation (4); the results are shown in figure 5. Figure 5 can be interpreted as showing the relative quality of a Carnot refrigerator compared with a thermoelectric refrigerator at each temperature.

* Note that the method presented can be applied for Z any reasonable function of temperature.

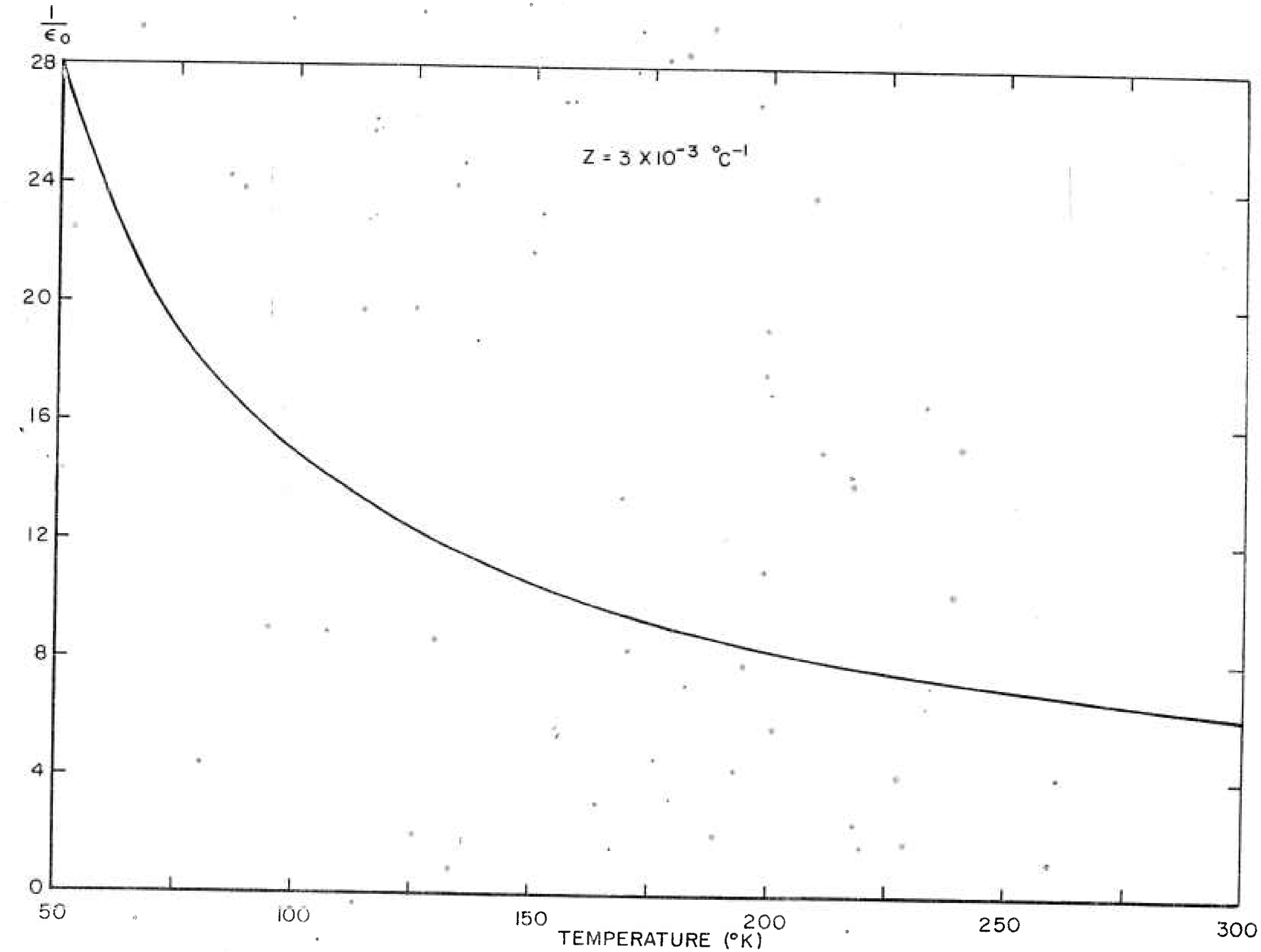

Figure 5 [illegible] of temperature for $Z = 3 \times 10^{-3}$ °C^{-1}

If we select an R of 1.25, ϵ_o will not vary excessively during any one interval down to about 50° K. This will give n a value of 8 for the temperature range from 300° down to 50° K. The limiting values of T_i for the intervals and the corresponding values of $1/\epsilon_o$ are shown in table 1.

Table 1. The Values of Temperature Intervals (T_i) and the Corresponding Values of ϵ_o at Specified Temperatures

T_i	Degrees (K)	Geometric mean of the temperatures*	$1/\epsilon_o$
T_h	300		
		268	6.7
T_7	240		
		214	7.8
T_6	192		
		172	9.3
T_5	154		
		137	11.4
T_4	123		
		110	14.0
T_3	98		
		88	16.6
T_2	79		
		70	20.4
T_1	63		
		56	25.4
T_c	50		

* The geometric mean of the temperature interval is defined as $T\ mean = \sqrt{T_i \cdot T_{i-1}}$.

Substituting the values in equation (22a) gives

$$\frac{Q_h}{Q_c} = (1.25)^{111.6} = 6.7 \times 10^{10} .$$

The values for cooling to the intermediate temperatures are plotted in figure 6, T_h is kept fixed at 300° K while T_c takes the values shown. The solution is shown in terms of Q_h/Q_c. From equation (7a) we see that, for values of Q_h/Q_c much greater than one, this equals W/Q_c which is the amount of electric power that must be supplied per amount of heat removed. Two important points about the form of dependence should be noticed. First, that for temperatures below 100° K the value of Q_h/Q_c rapidly becomes so large as to preclude the possibilities of cooling to this temperature region with any reasonable size device. Secondly, the amount of power needed to cool a small device to 200° K is comparatively negligible.

For comparison, the same calculation was repeated with n equal to 4 and R equal to 1.56. The results of this calculation are also shown. The close agreement obtained for the different values of n indicates that selecting the value of ϵ_o at the geometric mean of the temperature interval is a good method of averaging when there is a moderate change of ϵ_o during the interval. Q_h/Q_c for a single-stage cooler is also plotted for comparison, using equation (2).

The significance of figure 6 is more readily apparent if it is used to calculate the minimum power required to remove 50 mw of heat (a reasonable value for cooling a small infrared cell) as a function of T_c when T_h is fixed at 300° K. The results of this calculation are shown in figure 7. The low power consumption down to 200° K followed by a rapid increase in power consumption below that is easily seen. The equivalent curves for some other values of Z are shown also.

The general form of the results shown in figures 6 and 7 is not greatly affected by any reasonable temperature dependence of ϵ_o. This has been verified for dependence of the type $Z = T^a$, where a has values from +2 to -2.

The specific values of temperature and power, however, are functions of ϵ_o. A low temperature limit for a feasible thermoelectric refrigerator might be approximated as the value of T_c which will give a COP of 0.01 in equation (21). The results of this assumption are shown in figure 8. The lowest temperature obtainable in a single-stage device (eq 1) is shown for comparison. An actual multistage device operating as indicated by figure 8 would, of course, have a COP less than 0.01, since it should be remembered that the calculations performed here are highly idealized. No account has been taken of electrical contact resistance or thermal impedance between stages. In addition, heat leakage through the insulation will be

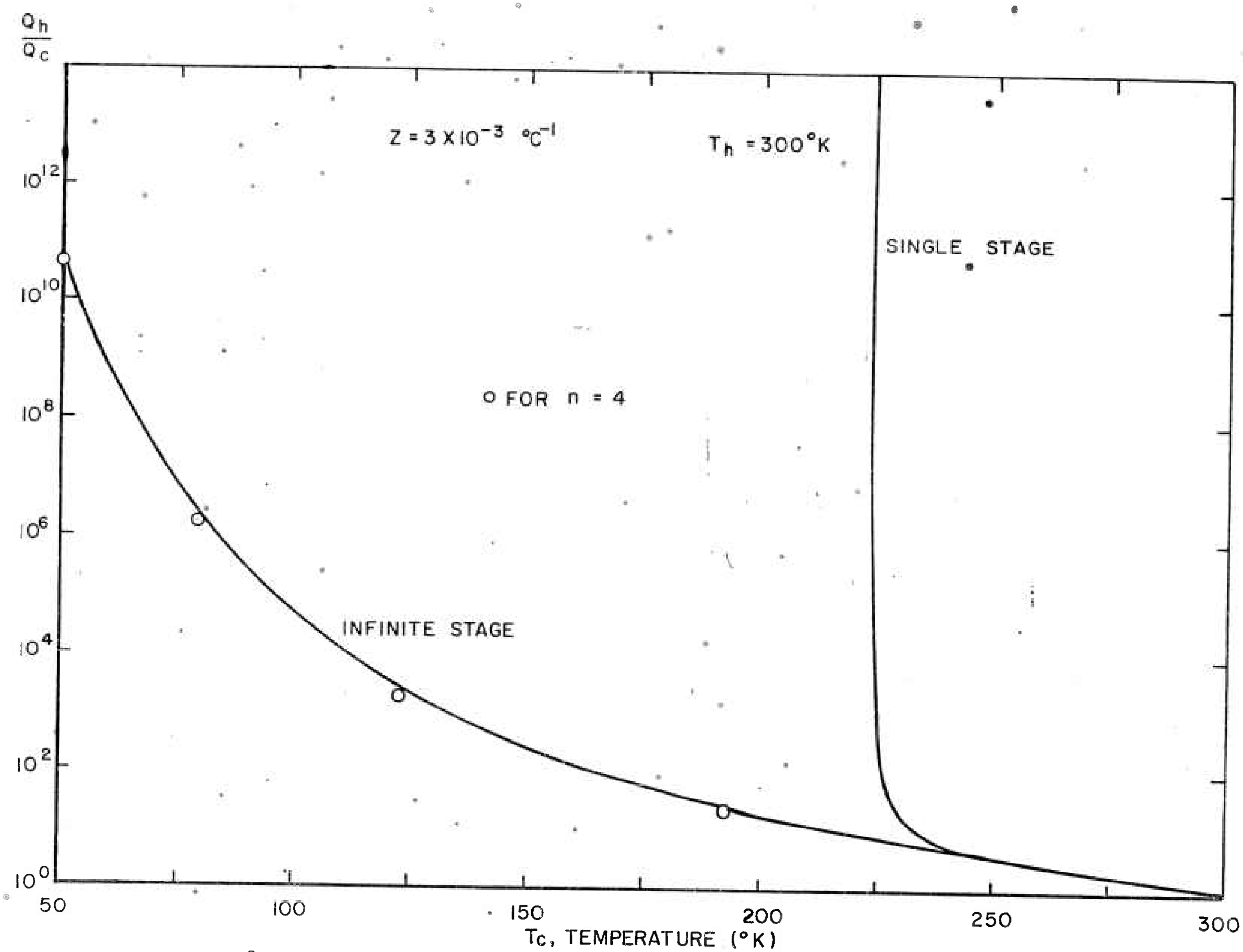

Figure 6. $\frac{\dot{Q}_h}{\dot{Q}_c}$ for an infinite-stage thermoelectric refrigerator as a function of temperature when T_h is fixed as 300°K.

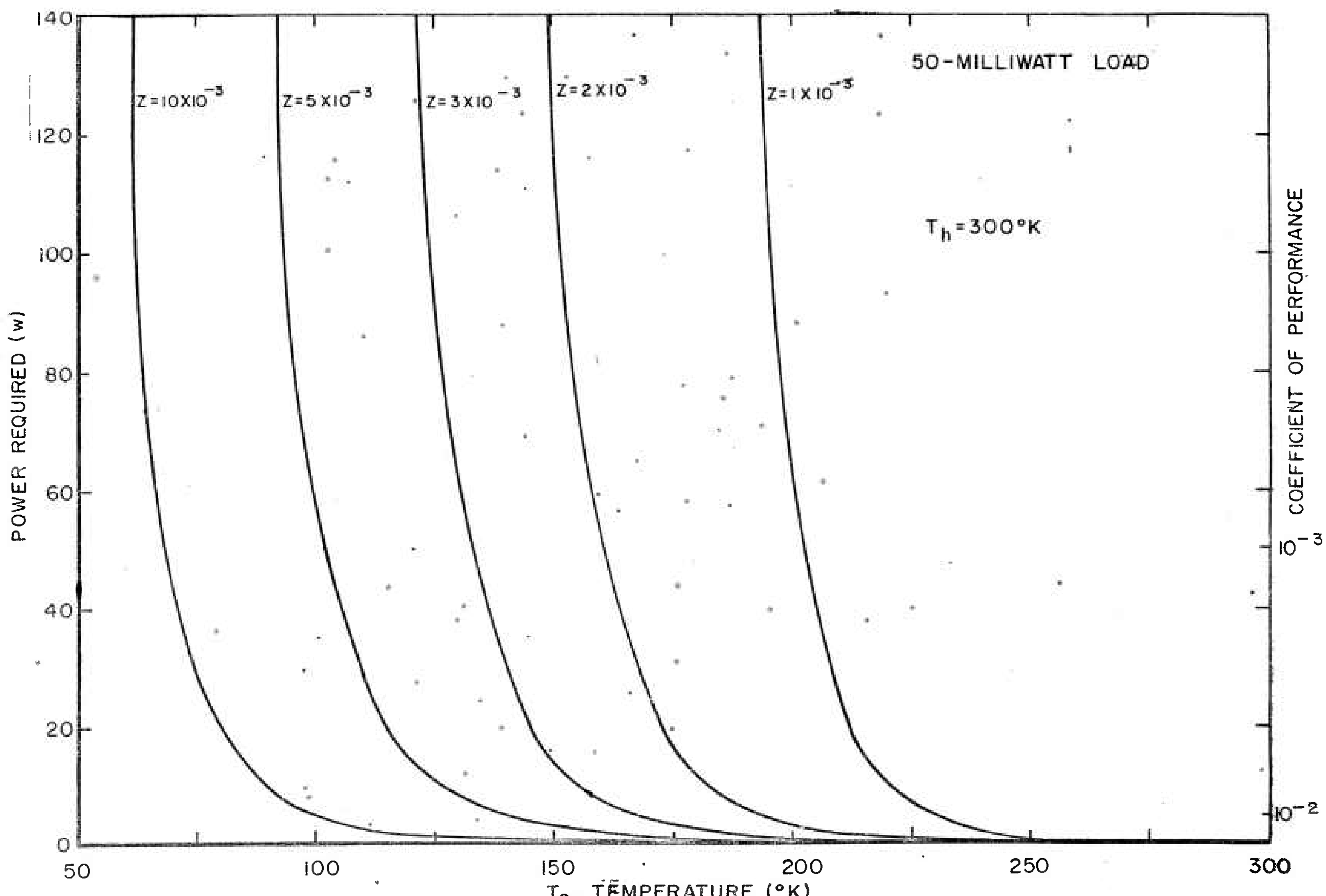

Figure 7. The minimum power required by a multistage thermoelectric refrigerator to remove 500 milliwats of heat (equivalent to an infrared cell) shown as a function of the cold-junction temperature when the hot-junction temperature is fixed at 300°K.

important in any practical device. Also a device with a finite number of stages must have a COP less than that shown in figure 8. These factors, however, will not change the general form of the power-temperature relationship.

Heat leakage and contact resistance will result in effectively lowering the value of ϵ_o. The result of thermal impedance between stages will be an increase in the effective value of the ratio R so that

$$R^n > \frac{T_h}{T_c} .$$

The effective value of R and ϵ_o can be inserted in equation (20) and the results calculated as before.* Depending on the values of the loss parameters, the temperature reductions shown in figures 5 and 6 will be reduced approximately 10 to 30 percent in most cases.

6. PRACTICAL APPROXIMATION TO INFINITE STAGING

Figure 6 shows that for moderate temperature difference the power required by a single stage approaches that of an infinite cascade. In figure 9 the data in figure 6 are replotted to better illustrate this. Therefore, for a temperature reduction of less than 25° or 30° C from 300° K and a Z value of 3×10^{-3} $^{o}C^{-1}$

$$\left(\frac{Q_h}{Q_c}\right)_{\infty \text{ stage}} \simeq \left(\frac{q_h}{q_c}\right)_{1 \text{ stage}} \qquad (23)$$

This is equivalent to $\epsilon_o \simeq \epsilon$ for this temperature interval. If a finite cascade is now constructed over a large temperature interval, each stage of which obeys equation (23) the resulting COP will closely approximate that for an infinite cascade over that interval. Since

$$\left(\frac{Q_h}{Q_c}\right)_{\infty} = \left(\frac{Q_1}{Q_c}\right)_{\infty}\left(\frac{Q_2}{Q_1}\right)_{\infty} \cdots \left(\frac{Q_h}{Q_{n-1}}\right)_{\infty} \qquad (24)$$

* In some cases the effect of the loss parameter might be approximated by using an "effective" value of Z. For example, a device built with material where Z is 3×10^{-3} $^{o}C^{-1}$ over the temperature interval covered might have a performance approximating that shown for $Z = 2 \times 10^{-3}$ $^{o}C^{-1}$ in figure 7.

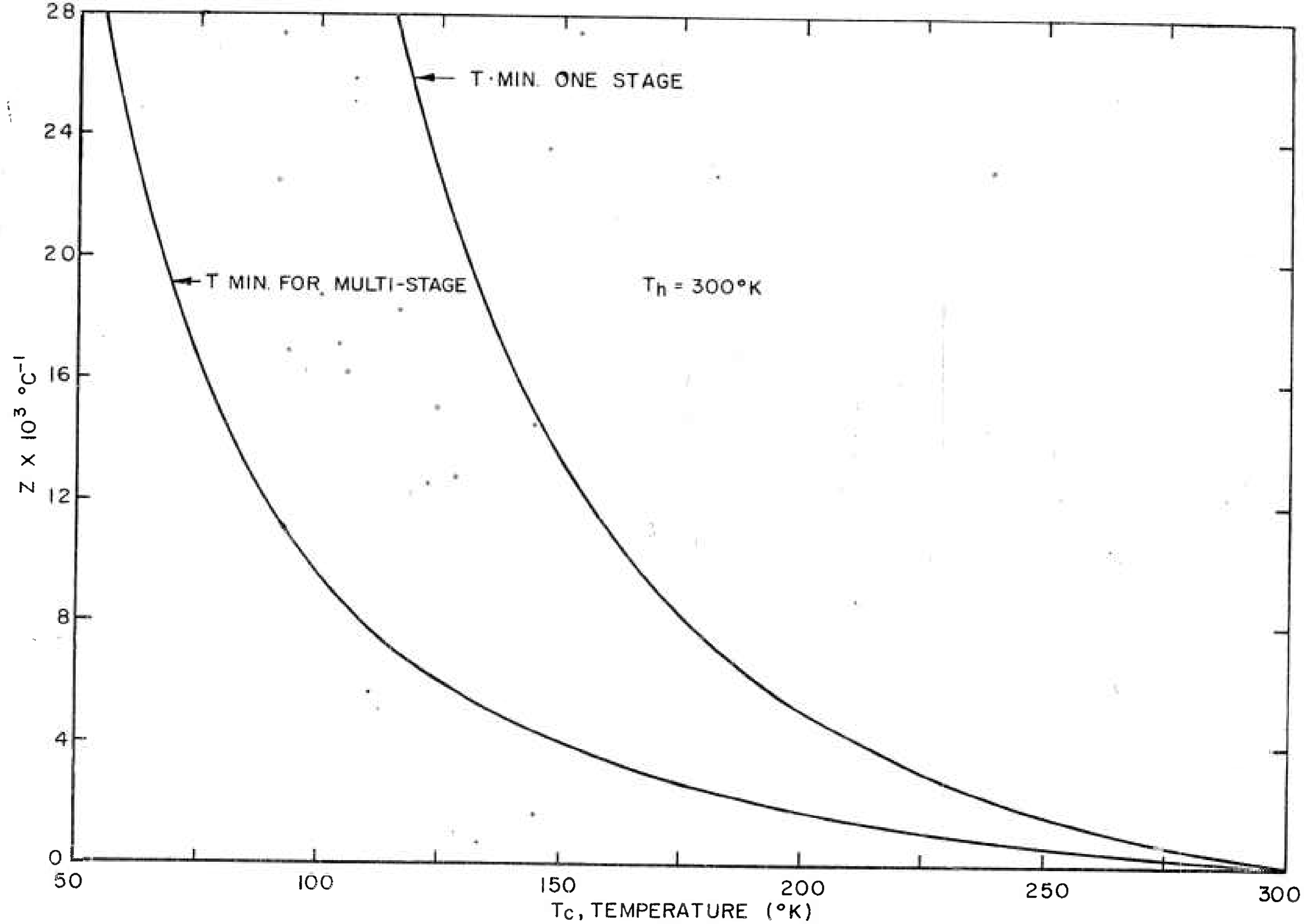

Figure 8. The approximate minimum temperature that can be reached by a feasible multistage thermoelectric refrigerator shown as a function of the figure of merit, Z.

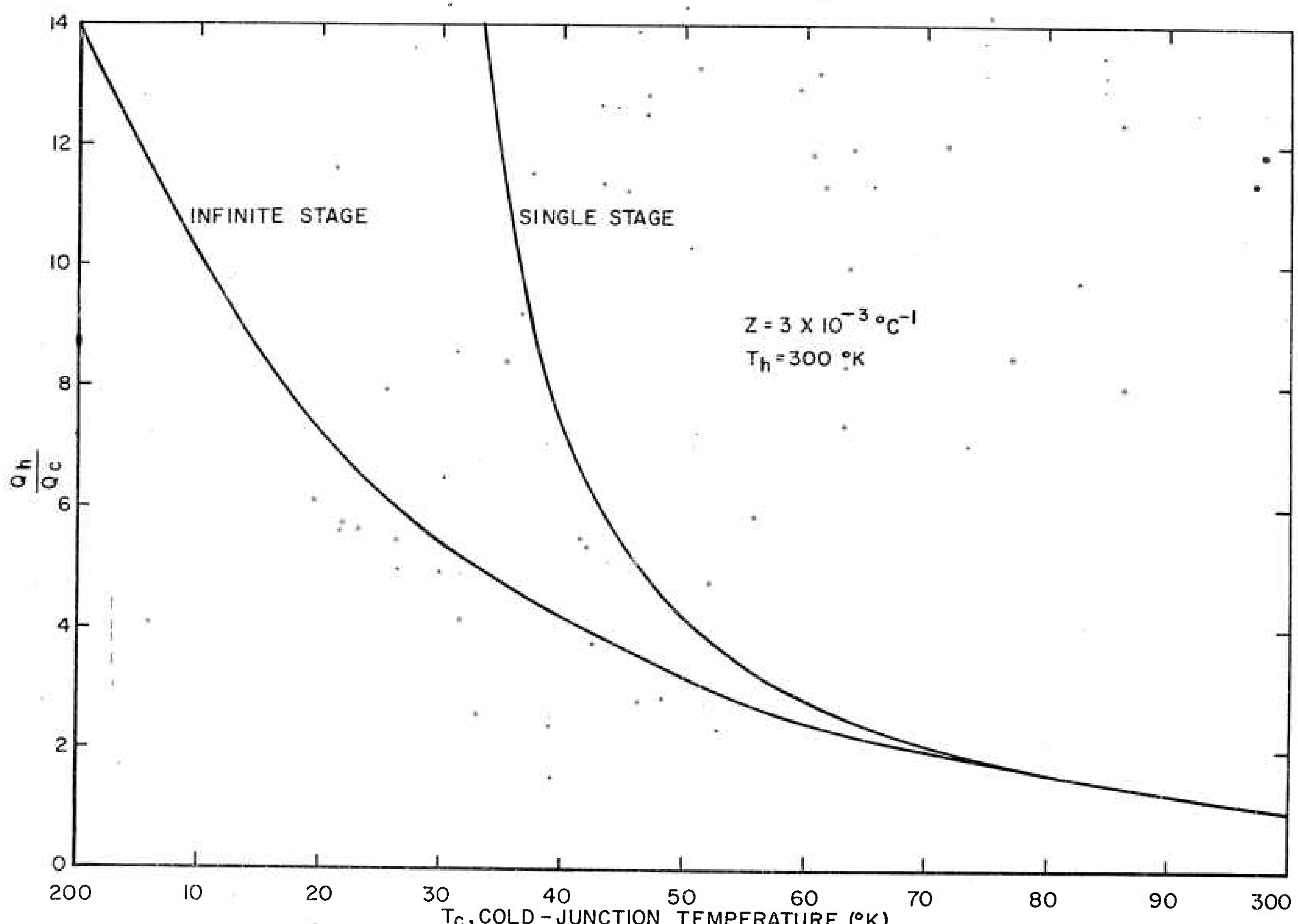

Figure 9. $\frac{Q_h}{Q_c}$ shown as a function of temperature for a hot-junction fixed at 300°K.

$$\left(\frac{Q_h}{Q_c}\right)_{finite} = \left(\frac{q_1}{Q_c}\right)_1 \left(\frac{q_2}{q_1}\right)_1 \cdots \left(\frac{Q_h}{q_{n-1}}\right)_1 , \tag{25}$$

and

$$\left(\frac{q_i}{q_{i-1}}\right)_1 \simeq \left(\frac{Q_i}{Q_{i-1}}\right)_\infty , \tag{25a}$$

$$\left(\frac{Q_h}{Q_c}\right)_\infty \simeq \left(\frac{Q_h}{Q_c}\right)_{finite} \tag{26}$$

Equation (26) is true even for a small number of stages n, provided the temperature differential per stage is moderate. As a matter of fact, the approximation breaks down if the total temperature differential is extended too greatly by an excessive number of stages, even if each has only a moderate temperature differential.

Figure 10 shows the relationship between the coefficients of performance and the temperature differences both for single and infinite stages. (Reference 7 gives a calculation for the special case of Zt and ϵ_o equal to zero.) The temperature difference is expressed in terms of the maximum obtainable with a single-stage cooler, cooling down from a fixed temperature t_h. This value is given by

$$\Delta t_{max} = \frac{(1 + Z\,t_h) - (2Z\,t_h + 1)^{\frac{1}{2}}}{Z} \tag{27}$$

If Z is a strong function of temperature, an average value of Z must be used, since the exact relationships are somewhat dependent on the value of $Z\,t$ or ϵ_o. The relationships are shown for $Z\,t = 1$ or $\epsilon_o = 0.17$ and $Z\,t = 0$ or $\epsilon_o = 0$. Intermediate values of these parameters give intermediate results. The result for a Carnot refrigerator, $Z\,t$ equal to infinity, is shown for comparison. From this, it can be seen that the temperature intervals covered by each stage of a finite cascade can be about $0.4\ \Delta t_{max}$ and still approximately satisfy the relationship given in equation (23). The value of $0.4\ \Delta t_{max}$ must be recalculated for each stage, using its own hot junction temperature.

If this approximation is applied to the limits of feasibility assumed in figure 8, it appears that it would never be desirable to construct a device with more than about six stages, even without considering

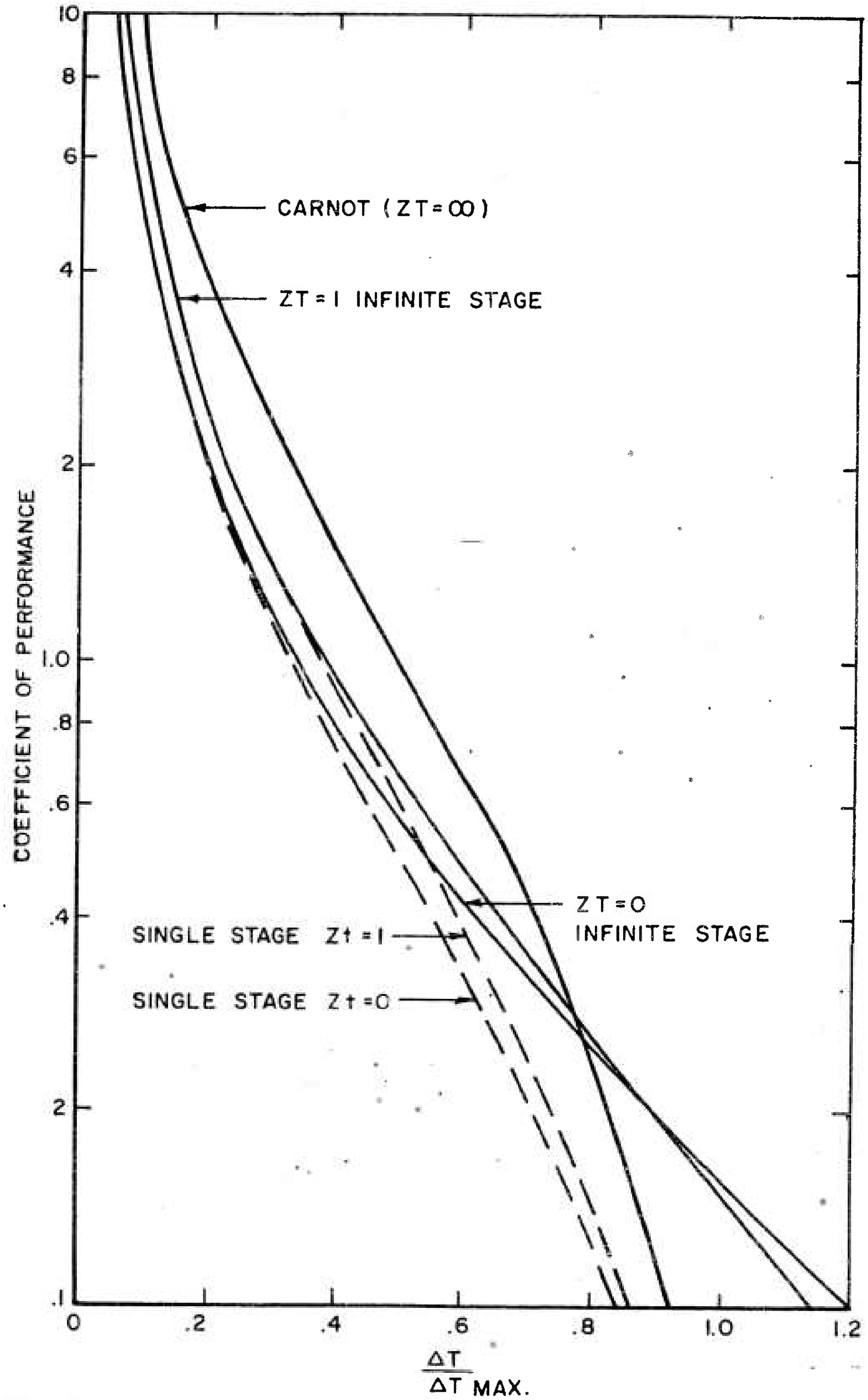

Figure 10. The maximum value of the coefficient of performance that can be obtained with single-and-infinite-stage refrigerator shown as a function of the parameter $\Delta T/\Delta T_{max}$.

constructional complexities. The increase in power requirement to make each stage $1/2\ \Delta t_{max}$ is not excessive if there are not too many stages. This would have the advantage of approximately maximizing the amount of heat removed per unit of material when equation (2) is satisfied. (This is derived by approximating equation 17 from page 99 of reference 1, as $I_o = \frac{I_{max}}{\Delta t_{max}} \Delta t$, then inserting this value of I_o in equation 9, page 97 and maximizing Q as a function of Δt.)

Figure 9 can also be used to approximate the ratio of heat removal capacity from stage to stage needed in a finite cascade. Using the relationship

$$1 + \frac{1}{cop} = \frac{q_h}{q_c}$$

for Δt's equal to 0.4 to 0.5 Δt_{max} gives a value for q_h/q_c of 2.0 to 3.0. Since the rejected heat of each stage is the input to the next stage, this is also the ratio of heat removal capacity needed. If the material parameters are not a rapid function of temperature, the amount of material needed in successive stages will have nearly the same ratio providing the length of the elements is kept constant.

7. SUMMARY & CONCLUSION

An easily applied method for calculating the maximum performance obtainable from thermoelectric devices has been presented. The maximum performance is obtained by the use of infinite staging. It was shown, however, that performance closely approximating that obtainable by infinite staging can be achieved by the use of a small number of stages: one or, at most, two in the case of a generator and several in the case of a refrigerator.

The application of infinite-staging analysis to a thermoelectric generator results in no significant new results compared with the conventional single-stage analysis. In the case of a thermoelectric refrigerator, however, results are developed that are important in the design of a practical device. These are:

(1) For temperature differences up to 0.4 of that obtainable with a single-stage device (24 to 30° C at present), a single-stage device will have a coefficient of performance closely approximating that of a multistage device.

(2) While, in principle, a multistage device could be designed to produce nearly any temperature reduction desired, in practice, it will be extremely difficult to build one that can produce twice the reduction obtainable with a single-stage unit. When present day materials are employed, this corresponds to a maximum temperature reduction of about 135° C.

(3) Even allowing for major advances in material quality (let us say to $Z = 10 \times 10^{-3}$), it does not seem probable that a thermoelectric device that could produce cryogenic temperature would be feasible.

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A simplified method of calculating the performance limits of multistage thermoelectric devices is presented and the results obtained for the multistage devices are compared with those for single-stage devices. The results indicate that at present there is little advantage in using more than one stage in a thermoelectric generator. In the case of a thermoelectric refrigerator, however, a small feasible multistage device should be able to produce temperature reduction about twice as great as that obtainable with a single-stage device. Using presently available materials, this corresponds to a maximum temperature reduction approximating 135°C for the multistage device. It is also shown that even if materials are substantially improved, it will probably never be feasible to construct a thermoelectric refrigerator that can cool to cryogenic temperatures while operating from room temperature ambient.

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